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**Web Resources**

How To Solve Quadratic Equations: www.mathwarehouse.com/quadratic/solve-quadratic-equation.php
Completing the Square: www.mathwarehouse.com/quadratic/completing-the-square-math.php



Graph Paper Maker (free): www.mathworksheetsgo.com/paper/

Online Graphing Calculator :www.mathworksheetsgo.com/calculator/

# Solve Quadratic Equations by Completing the Square

## I. Model Problems

In the following examples you will solve quadratic equations by completing the square.

*Example 1*: **Solve $x^2 + 8x - 6 = 0$ over the set of Reals.**

Write down the equation.

Isolate $ax^2 + bx$ part of the equation using additive inverse.

$$x^2 + 8x - 6 = 0$$
$$+6 \quad +6$$

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$$x^2 + 8x = 6$$

We know $(x + b)^2 = x^2 + 2bx + b^2$. If the right side of the equation is a perfect square, then $8 = 2b$ or $b = 4$. Then to complete the square we need to add the third term of the trinomial ($b^2$). In this case that is $4^2$ which equals 16. If we add that to the left side of the equation, we need to add it to the right side too.

$$x^2 + 8x + 16 \quad +16$$

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$$(x + 4)^2 = 22$$

Now take the square root of both sides and simplify the radical if needed.

$$x + 4 = \pm\sqrt{22}$$

Subtract four from both sides

$$-4 \quad -4$$

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Simplify.

$$x = -4 \pm \sqrt{22}$$

**The solution is: $x = -4 \pm \sqrt{22}$.**

*Example 2*: **Solve $2x^2 - 20x + 64 = 0$ over the set of Complex Numbers.**

Write down the equation.

$$2x^2 - 20x + 64 = 0$$

We want *a*, the coefficient of the $x^2$ term to be 1. Divide both sides by 2.

$$x^2 - 10x + 32 = 0$$

Isolate $ax^2 + bx$ part of the equation using additive inverse.

$$-32 \quad -32$$

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$$x^2 - 10x = -32$$

Complete the square. Half of $-10$ is $-5$, and $-5$ square is 25.

$$x^2 - 10x + 25 \quad +25$$

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$$(x - 5)^2 = -7$$

Now take the square root of both sides and simplify the radical if needed. In this case we are solving over the set of Complex numbers so we have an imaginary solution. (If we were solving over the set of Reals there would be no Real solution.)

$$x - 5 = \pm i\sqrt{7}$$

Add five to both sides

$$+5 \quad +5$$

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Simplify.

$$x = 5 \pm i\sqrt{7}$$

**The solution is: $x = 5 \pm i\sqrt{7}$.**

**II. Practice solving quadratics by completing the square.**

**Solve over the set of Reals.**

1. $x^2 + 6x - 9 = 0$
2. $x^2 - 10x + 15 = 0$
3. $x^2 + 16x + 24 = 0$
4. $x^2 - 20x + 4 = 0$
5. $x^2 + 24x - 16 = 0$
6. $x^2 + 6x + 9 = 0$
7. $2x^2 + 8x + 20 = 0$
8. $3x^2 + 12x - 5 = 7$
9. $5x^2 - 18x + 4 = 12x + 39$
10. $6x^2 - 11x + 20 = 2 - 95x$

**Solve over the set of Complex Numbers.**

11. $4x^2 - 24x + 100 = 0$
12. $7x^2 - 20x + 4 = 8x - 10$
13. $x^2 + 3x - 3 = 0$
14. $x^2 + 5x + 7 = 0$
15. $-2x^2 + 6x + 12 = 0$
16. $3x^2 - 6x + 15 = 0$
17. $2x^2 - 24x + 80 = 0$
18. $3x^2 + 9x + 18 = 42 - 6x$
19. $x^2 - 14x + 49 = -18$
20. $14 = 3x^2 + 21x + 2$

**III. Challenge Problems**

21. Solve $x^4 - 18x^2 + 17 = 0$ over the set of Complex Numbers.
22. Solve $x^4 + 20x^2 - 44 = 0$ over the set of Complex Numbers.
23. The vertex form of the equation of a parabola is $y = a(x - h)^2 + k$. Complete the square to rewrite $y = x^2 + 4x + 5$ in vertex form.
24. The vertex form of the equation of a parabola is $y = a(x - h)^2 + k$. Complete the square to rewrite $y = x^2 - 8x - 6$ in vertex form.
25. Solve for *x* by completing the square.
    $0 = ax^2 + bx + c$

**IV. Answer Key**

1. $x = -3 \pm 3\sqrt{2}$
2. $x = 5 \pm \sqrt{10}$
3. $x = -8 \pm 2\sqrt{10}$
4. $x = 10 \pm 4\sqrt{6}$
5. $x = -12 \pm 4\sqrt{10}$
6. $x = -3 \pm \sqrt{2}$
7. $\emptyset$
8. $x = -2 \pm 2\sqrt{2}$
9. $x = 4 \pm \sqrt{23}$
10. $x = -7 \pm \sqrt{46}$
11. $x = 3 \pm i$
12. $x = 2 \pm \sqrt{2}$
13. $x = \frac{-3 \pm \sqrt{21}}{2}$
14. $x = \frac{-5 \pm i\sqrt{3}}{2}$
15. $x = \frac{3 \pm \sqrt{33}}{2}$
16. $x = 1 \pm 2i$
17. $x = 6 \pm 2i$
18. $x = \frac{-5 \pm \sqrt{57}}{2}$
19. $x = 7 \pm 3i\sqrt{2}$
20. $x = \frac{-7 \pm \sqrt{65}}{2}$
21. $x = \pm\sqrt{17},\ \pm 1$
22. $x = \pm\sqrt{2}, \pm i\sqrt{22}$
23. $y = (x+2)^2 - 1$
24. $y = (x-4)^2 - 22$
25. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$